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Non-supersymmetric meta-stable vacua in SU(N) SQCD with adjoint matter

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Abstract: We investigate models of SU(N) SQCD with adjoint matter and non trivial mesonic deformations. We apply standard methods in the dual magnetic theory and we find meta-stable supersymmetry breaking vacua with arbitrary large lifetime. We comment on the difference with known models.

Keywords: Supersymmetry Breaking, Supersymmetry and Duality, Supersymmetric gauge theory.

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Contents

1. Introduction

Long living meta-stable vacua breaking supersymmetry exist in classes of $\mathcal{N} = 1$ gauge theories of the SQCD type with massive fundamental matter [1, 2].

The novelty of the approach of [2] relies on theories for which Seiberg-like duality exists i.e. (electric) theories which are asymptotically free in the ultraviolet and strongly coupled in the infrared, where the physics can be described in terms of weakly coupled dual (magnetic) theories (for reviews see [3, 4]). In the region of small fields this dual description can be studied as a model of pure chiral fields. Supersymmetry is broken by the rank condition [2], i.e. not all the F-term conditions can be satisfied. Roughly speaking the next step is to recover, in this magnetic infrared, a generalized chiral O'Raifeartaigh model with supersymmetry breaking vacua.

These non supersymmetric vacua have typically classical flat directions which can be lifted by quantum corrections. In [2] it has been proved that such corrections generate positive mass terms for the pseudo-moduli leading to long lived metastable vacua. These facts should be tested in different supersymmetric theories. Some generalization have already appeared upholding the notion that such phenomenon is rather generic [5 − 7]. The relative stability of the vacua is a rather delicate issue. Remarks about the corresponding string configurations corroborating the stability analysis have also appeared [8 − 11].

In this paper we study theories with adjoint chiral fields with cubic superpotential à la KSS Such superpotentials generate a further meson in the dual magnetic theory: this might produce several pseudo-Goldstone excitations and jeopardize the 1-loop stability of the non supersymmetric vacua. There must be enough F and/or D equations to give tree level masses. A viable model, of string origin, with two gauge groups has been presented in [6].

We consider a theory with one gauge group $\mathrm{SU}(N_c)$ and two massive electric adjoint fields, where the most massive one gets integrated out. This amounts to add a massive mesonic deformation in the dual theory. This avoids dangerous extra flat directions which cannot be stabilized at 1-loop. A discussion of the possible interpretation via D-brane configurations can be found in [15, 16].

In the study of the magnetic dual theory we find a tree-level non supersymmetric vacuum which is stabilized by quantum corrections; we show that this is a metastable state that decays to a supersymmetric one after a parametrically long time. A landscape of non supersymmetric metastable vacua, present at classical level, disappears at quantum level. Differently from [2, 5, 6] in our model there is no $\mathrm{U}(1)_R$ symmetry and our minimum will not be at the origin of the field space, making our computation much involved. We present most of our results graphically, giving analytic expressions in some sensible limits. We follow the computational strategy of [2].

In section 2 we recall some basic elements of the KSS duality and introduce the model that we consider through the paper. In section 3 we solve the D and F equations finding an energy local minimum where supersymmetry is broken by a rank condition. In section 4 we compute the 1-loop effective potential around this vacuum and find that it is stabilized by the quantum corrections. In section 5 we restore supersymmetry by non perturbative gauge dynamics and recover supersymmetric vacua. Using this result we estimate the lifetime of our metastable vacuum in section 6.

2. $\mathcal{N} = 1$ SQCD with adjoint matter

Here we introduce some useful elements about electric/magnetic duality for supersymmetric gauge theories with an adjoint field [12−14]. We consider $\mathcal{N} = 1$ supersymmetric $\mathrm{SU}(N_c)$ Yang Mills theory coupled to N_f massive flavours $(Q^i_\alpha, \tilde{Q}^{j\beta})$ in the fundamental and antifundamental representations of the gauge group $(\alpha, \beta = 1, \ldots N_c)$ and in the antifundamental and fundamental representations of the flavour group $(i, j = 1, \ldots N_f)$, respectively. We also consider a charged chiral massive adjoint superfield X^α_β with superpotential[1]

$$W_{el} = \frac{g_X}{3}\mathrm{Tr}X^3 + \frac{m_X}{2}\mathrm{Tr}X^2 + \lambda_X \mathrm{Tr}X \tag{2.1}$$

where λ_X is a Lagrange multiplier enforcing the tracelessness condition $\mathrm{Tr}X = 0$. The Kahler potential for all the fields is taken to be canonical. This theory is asymptotically free in the range $N_f < 2N_c$ and it admits stable vacua for $N_f > \frac{N_c}{2}$ [13].

The dual theory [12−14] is $\mathrm{SU}(2N_f - N_c \equiv \tilde{N})$ with N_f magnetic flavours $(q, \tilde{q})$, a magnetic adjoint field Y and two gauge singlets build from electric mesons ($M_1 = Q\tilde{Q}$, $M_2 = QX\tilde{Q}$), with magnetic superpotential

$$W_{\mathrm{magn}} = \frac{\tilde{g}_Y}{3}\mathrm{Tr}Y^3 + \frac{\tilde{m}_Y}{2}\mathrm{Tr}Y^2 + \tilde{\lambda}_Y \mathrm{Tr}Y - \frac{1}{\mu^2}\mathrm{tr}\,\left(\frac{\tilde{m}_Y}{2}M_1 q\tilde{q} + \tilde{g}_Y M_2 q\tilde{q} + \tilde{g}_Y M_1 q Y \tilde{q}\right) \tag{2.2}$$

where the relations between the magnetic couplings and the electric ones are

$$\tilde{g}_Y = -g_X, \qquad \tilde{N}\tilde{m}_Y = N_c m_X. \tag{2.3}$$

[1](Tr) means tracing on the color indices, while (tr) on the flavour ones.

The intermediate scale μ takes into account the mass dimension of the mesons in the dual description. The matching between the microscopic scale (Λ) and the macroscopic scale ($\tilde{\Lambda}$) is

$$\Lambda^{2N_c-N_f}\tilde{\Lambda}^{2\tilde{N}-N_f} = \left(\frac{\mu}{g_X}\right)^{2N_f} . \tag{2.4}$$

We look for a magnetic infrared free regime in order to rely on perturbative computations at low energy. The b coefficient of the beta function is $b = (3\tilde{N} - N_f) - \tilde{N}$, negative for $N_f < \frac{2}{3}N_c$ and so we will consider the window for the number of flavours

$$\frac{N_c}{2} < N_f < \frac{2}{3}N_c \qquad \Rightarrow \qquad 0 < 2\tilde{N} < N_f \tag{2.5}$$

where the magnetic theory is IR free and it admits stable vacua.

2.1 Adding mesonic deformations

We now add to the electric potential (2.1) the gauge singlet deformations

$$W_{el} \to W_{el} + \Delta W_{el} \qquad \Delta W_{el} = \lambda_Q \,\mathrm{tr}\, QX\tilde{Q} + m_Q \,\mathrm{tr}\, Q\tilde{Q} + h\,\mathrm{tr}\,(Q\tilde{Q})^2 \tag{2.6}$$

The first two terms are standard deformations of the electric superpotential that don't spoil the duality relations (e.g. the scale matching condition (2.4)) [14]. The last term of (2.6) can be thought as originating from a second largely massive adjoint field Z in the electric theory with superpotential

$$W_Z = m_Z \mathrm{Tr} Z^2 + \mathrm{Tr}\, ZQ\tilde{Q} \tag{2.7}$$

and which has been integrated out [15 − 17]. The mass m_Z has to be considered larger than Λ_{2A}, the strong scale of the electric theory with two adjoint fields. This procedure leads to the scale matching relation

$$\Lambda_{2A}^{N_c-N_f} = \Lambda_{1A}^{2N_c-N_f} m_Z^{-N_c} \tag{2.8}$$

where Λ_{2A} and Λ_{1A} are the strong coupling scales before and after having integrated out the adjoint field Z, i.e. with two or one adjoint fields respectively.

The other masses in this theory have to be considered much smaller than the strong scale $\Lambda_{2A} \gg m_Q, m_X$. This forces, via (2.8), the scale Λ_{1A} and the masses to satisfy the relations

$$\frac{m_Q m_Z}{\Lambda_{1A}^2} \ll 1 \qquad\qquad \frac{m_X m_Z}{\Lambda_{1A}^2} \ll 1 \tag{2.9}$$

We will work in this range of parameters in the whole paper, translating these inequalities in the dual (magnetic) context.

We also observe that in (2.8) the coefficient b of the beta function for the starting electric theory with two adjoint fields is $b = N_c - N_f$ and the theory is asymptotically free for $N_f < N_c$. This range is still consistent with our magnetic IR free window (2.5). The dimensional coupling h in our effective theory (2.6) results $h = \frac{1}{m_Z}$ so it must be thought

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as a small deformation. In analogy with [17][2] we can suppose that when h is small the duality relations are still valid and obtain the full magnetic superpotential

$$W_{\rm magn} = \frac{\tilde{g}_Y}{3}{\rm Tr}Y^3 + \frac{\tilde{m}_Y}{2}{\rm Tr}Y^2 + \tilde{\lambda}_Y {\rm Tr}Y - \frac{1}{\mu^2}{\rm tr}\left(\frac{\tilde{m}_Y}{2}M_1 q\tilde{q} + \tilde{g}_Y M_2 q\tilde{q} + \tilde{g}_Y M_1 q Y \tilde{q}\right)$$
$$+\lambda_Q \;{\rm tr}\; M_2 + m_Q \;{\rm tr}\; M_1 + h \;{\rm tr}\; (M_1)^2 \tag{2.10}$$

For this dual theory the scale matching relation is the same as (2.4) with $\Lambda \equiv \Lambda_{1A}$ defined in (2.8).

We consider the free magnetic range (2.5), where the metric on the moduli space is smooth around the origin [2]. The Kahler potential is thus regular and has the canonical form

$$K = \frac{1}{\alpha_1^2\Lambda^2}{\rm tr}\; M_1^\dagger M_1 + \frac{1}{\alpha_2^2\Lambda^4}{\rm tr}\; M_2^\dagger M_2 + \frac{1}{\beta^2}{\rm Tr}Y^\dagger Y + \frac{1}{\gamma^2}({\rm tr}\; q^\dagger q + {\rm tr}\; \tilde{q}^\dagger \tilde{q}) \tag{2.11}$$

where $(\alpha_i, \beta, \gamma)$ are unknown positive numerical coefficients.

3. Non supersymmetric meta-stable vacua

We solve the equations of motion for the chiral fields of the macroscopic description (2.10). We will find a non supersymmetric vacuum in the region of small fields where the SU$(\tilde{N})$ gauge dynamics is decoupled. The gauge dynamics becomes relevant in the large field region where it restores supersymmetry via non perturbative effects (see section5).

We rescale the magnetic fields appearing in (2.10) in order to work with elementary fields with mass dimension one. We then have a $\mathcal{N} = 1$ supersymmetric SU$(\tilde{N})$ gauge theory with N_f magnetic flavours $(q, \tilde{q})$, an adjoint field Y, and two gauge singlet mesons M_1,M_2, with canonical Kahler potential. The superpotential, with rescaled couplings, reads

$$W_{\rm magn} = \frac{g_Y}{3}{\rm Tr}Y^3 + \frac{m_Y}{2}{\rm Tr}Y^2 + \lambda_Y {\rm Tr}Y + {\rm tr}\;(h_1 M_1 q\tilde{q} + h_2 M_2 q\tilde{q} + h_3 M_1 q Y \tilde{q})$$
$$-h_1 m_1^2 \;{\rm tr}\; M_1 - h_2 m_2^2 \;{\rm tr}\; M_2 + m_3 \;{\rm tr}\; M_1^2 \tag{3.1}$$

where the rescaled couplings in (3.1) are mapped to the original ones in (2.10) via

$$h_1 = -\frac{\tilde{m}_Y}{2\mu^2}(\alpha_1\Lambda)\,\gamma^2 \qquad h_2 = -\frac{\tilde{g}_Y}{\mu^2}(\alpha_2\Lambda^2)\,\gamma^2 \qquad h_3 = -\frac{\tilde{g}_Y}{\mu^2}(\alpha_1\Lambda)\,\gamma^2\beta$$

$$h_1 m_1^2 = -m_Q\,\alpha_1\,\Lambda \qquad h_2 m_2^2 = -\lambda_Q\,\alpha_2\,\Lambda^2 \qquad m_3 = h(\alpha_1\Lambda)^2 \tag{3.2}$$

We can choose the magnetic quarks $q, \tilde{q}^T$ (which are $N_f \times \tilde{N}$ matrices) to solve the D equations as

$$q = \begin{pmatrix} k \\ 0 \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} \tilde{k} \\ 0 \end{pmatrix} \tag{3.3}$$

where $k, \tilde{k}$ are $\tilde{N} \times \tilde{N}$ diagonal matrices.

[2] Where it was done in the context of Seiberg duality.

We impose the F equations of motion for the superpotential (3.1)

$$\begin{aligned}
F_{\lambda_Y} &= \mathrm{Tr} Y = 0 \\
F_Y &= g_Y Y^2 + m_Y Y + \lambda_Y + h_3 M_1 q\tilde{q} = 0 \\
F_q &= h_2 M_2 \tilde{q} + h_1 M_1 \tilde{q} + h_3 M_1 Y \tilde{q} = 0 \\
F_{\tilde{q}} &= h_2 M_2 q + h_1 M_1 q + h_3 M_1 q Y = 0 \qquad (3.4) \\
F_{M_1} &= h_1 q\tilde{q} + h_3 q Y \tilde{q} - h_1 m_1^2 \delta_{ij} + 2 m_3 M_1 = 0 \qquad i,j = 1,\dots N_f \\
F_{M_2} &= h_2 q\tilde{q} - h_2 m_2^2 \delta_{ij} = 0 \qquad i,j = 1,\dots N_f \qquad (3.5)
\end{aligned}$$

Since we are in the range (2.5) where $N_f > \tilde{N}$ the equation (3.5) is the rank condition of [2]: supersymmetry is spontaneously broken at tree-level by these non trivial F-terms.

We can solve the first $\tilde{N}$ equations of (3.5) by fixing the product $k\tilde{k}$ to be $k\tilde{k} = m_2^2 \mathbf{1}_{\tilde{N}}$. We then parametrize the quarks vevs in the vacuum (3.3) with complex θ

$$q = \begin{pmatrix} m_2 e^{\theta}\, \mathbf{1}_{\tilde{N}} \\ 0 \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} m_2 e^{-\theta}\, \mathbf{1}_{\tilde{N}} \\ 0 \end{pmatrix} . \qquad (3.6)$$

The other $N_f - \tilde{N}$ equations of (3.5) cannot be solved and so the corresponding F-terms don't vanish ($F_{M_2} \neq 0$). However we can find a vacuum configuration which satisfies all the other F-equations (3.4) and the D-ones. We solve the equations (3.4) for M_1, Y and λ_Y and we choose Y to be diagonal, finding

$$\lambda_Y = \frac{h_3 h_1 m_2^2}{2m_3}\left(m_2^2 - m_1^2\right) - \frac{m_Y^2}{g_Y}\left(1 - \frac{h_3^2 m_2^4}{2 m_3 m_Y}\right)^2 \frac{n_1 n_2}{(n_1 - n_2)^2} \qquad (3.7)$$

where the integers (n_1, n_2) count the eigenvalues degeneracy along the Y diagonal, with $(n_1 + n_2 = \tilde{N})$

$$\langle Y \rangle = \begin{pmatrix} y_1 \mathbf{1}_{n_1} & 0 \\ 0 & y_2 \mathbf{1}_{n_2} \end{pmatrix} \qquad y_1 = -\frac{m_Y - \frac{h_3^2 m_2^4}{2m_3}}{g_Y} \frac{n_2}{n_1 - n_2} \qquad y_2 = \frac{m_Y - \frac{h_3^2 m_2^4}{2m_3}}{g_Y} \frac{n_1}{n_1 - n_2}$$

We choose the vacuum in which the magnetic gauge group is not broken by the adjoint field choosing $n_1 = 0$, so y_2 vanishes and $\langle Y \rangle = 0$. We observe that other choices for $\langle Y \rangle$ with $n_1 \neq 0 \neq n_2$ wouldn't change the tree-level potential energy of the vacua which is given only by the non vanishing F_{M_2}. This classical landscape of vacua will be wiped out by 1-loop quantum corrections.[3] In our case ($n_1 = 0$) we have

$$\langle M_1 \rangle = \begin{pmatrix} \frac{h_1}{2m_3}(m_1^2 - m_2^2)\, \mathbf{1}_{\tilde{N}} & 0 \\ 0 & \frac{h_1 m_1^2}{2m_3}\, \mathbf{1}_{N_f - \tilde{N}} \end{pmatrix} = \begin{pmatrix} p_1^A & 0 \\ 0 & p_1^B \end{pmatrix} \qquad (3.8)$$

The two non trivial blocks are respectively $\tilde{N}$ and $N_f - \tilde{N}$ diagonal squared matrices.

The $(q, \tilde{q})$ F equations fix the vev of the M_2 meson to be

$$\langle M_2 \rangle = \begin{pmatrix} -\frac{h_1^2}{2h_2 m_3}(m_1^2 - m_2^2)\, \mathbf{1}_{\tilde{N}} & 0 \\ 0 & \mathcal{X} \end{pmatrix} = \begin{pmatrix} p_2^A & 0 \\ 0 & \mathcal{X} \end{pmatrix} \qquad (3.9)$$

[3] This agrees with an observation in [6].

where the blocks have the same dimensions of M_1, with $\mathcal{X}$ undetermined at the classical level.

Since supersymmetry is broken at tree level by the rank condition (3.5) the minimum of the scalar potential is

$$V_{\mathrm{MIN}} = |F_{M_2}|^2 = (N_f - \tilde{N})|h_2 m_2^2|^2 = (N_f - \tilde{N})\, \alpha_2^2\, |\lambda_Q \Lambda^2|^2 \tag{3.10}$$

It depends on parameters that we can't compute from the electric theory (e.g. α_2); in any case we are only interested in the qualitative behaviour of the non supersymmetric state. The potential energy of the vacuum (3.10) doesn't depend on θ and $\mathcal{X}$; they are massless fields at tree level, not protected by any symmetry and hence are pseudo-moduli. Their fate will be decided by the quantum corrections.

Since there isn't any $\mathrm{U}(1)_R$ symmetry we don't expect the value of $\mathcal{X}$ in the quantum minimum to vanish. Indeed, computing the 1-loop corrections, we will find that in the quantum minimum the value of θ is zero while $\mathcal{X}$ will get a nonzero vev. This makes our metastable minimum different from the one discovered in [2, 5, 6] where the quantum corrections didn't give the pseudo-moduli a nonzero vev. Notice also that although we have many vevs different from zero in the non supersymmetric vacua they are all smaller than the natural breaking mass scale $|F_{M_2}|^{\frac{1}{2}} = |h_2 m_2^2|^{\frac{1}{2}}$.

4. 1-Loop effective potential

In this section we study the 1-loop quantum corrections to the effective potential for the fluctuations around the non supersymmetric vacuum selected in the previous section with $\langle Y \rangle = 0$. The aim is to estabilish the sign of the mass corrections for the pseudo-moduli $\mathcal{X}, \theta$. The 1-loop corrections to the tree level potential energy depend on the choice of the adjoint vev $\langle Y \rangle$: as a matter of fact they are minimized by the choice $\langle Y \rangle = 0$.

The 1-loop contributions of the vector multiplet to the effective potential vanish since the D equations are satisfied by our non supersymmetric vacuum configuration.

The 1-loop corrections will be computed using the supertrace of the bosonic and fermionic squared mass matrices built up from the superpotential for the fluctuations of the fields around the vacuum. The standard expression of the 1-loop effective potential is

$$V_{1-loop} = \frac{1}{64\pi^2} S\mathrm{Tr}\mathcal{M}^4 \log \frac{\mathcal{M}^2}{\Lambda^2} = \frac{1}{64\pi^2} \sum \left(m_B^4 \log \frac{m_B^2}{\Lambda^2} - m_F^4 \log \frac{m_F^2}{\Lambda^2} \right) \tag{4.1}$$

where the F contributions to the mass matrices are read from the superpotential W

$$m_B^2 = \begin{pmatrix} W^{\dagger ac} W_{cb} & W^{\dagger abc} W_c \\ W_{abc} W^{\dagger c} & W_{ac} W^{\dagger cb} \end{pmatrix} \qquad m_f^2 = \begin{pmatrix} W^{\dagger ac} W_{cb} & 0 \\ 0 & W_{ac} W^{\dagger cb} \end{pmatrix} \tag{4.2}$$

We parametrize the fluctuations around the tree level vacuum as

$$q = \begin{pmatrix} k e^{\theta} + \xi_1 \\ \phi_1 \end{pmatrix} \qquad \tilde{q}^T = \begin{pmatrix} k e^{-\theta} + \xi_2 \\ \phi_2 \end{pmatrix} \qquad Y = \delta Y \tag{4.3}$$

$$M_1 = \begin{pmatrix} p_1^A + \xi_3 & \phi_3 \\ \phi_4 & p_1^B + \xi_4 \end{pmatrix} \quad M_2 = \begin{pmatrix} p_2^A + \xi_5 & \phi_5 \\ \phi_6 & \mathcal{X} \end{pmatrix} \tag{4.4}$$

We expand the classical superpotential (3.1) up to trilinear order in the fluctuations $\phi_i, \xi_i, \delta Y$. Most of these fields acquire tree level masses, but there are also massless fields. Some of them are Goldstone bosons of the global symmetries, considering $\mathrm{SU}(\tilde{N})$ global, the others are pseudo-Goldstone bosons.

In this set up, ξ_1 and ξ_2 combine to give the same Goldstone and pseudo-Goldstone bosons as in [2]. Gauging the $\mathrm{SU}(\tilde{N})$ symmetry these Goldstone bosons are eaten by the vector fields, and the other massless fields, except $\theta+\theta^\star$, acquire positive masses from D-term potential as in [2]. Combinations of the ϕ_i fields give the Goldstone bosons related to the breaking of the flavour symmetry $\mathrm{SU}(N_f)\to\mathrm{SU}(\tilde{N})\times\mathrm{SU}(N_f-\tilde{N})\times\mathrm{U}(1)$. The off diagonal elements of the classically massless field $\mathcal{X}$ are Goldstone bosons of the $\mathrm{SU}(N_f-\tilde{N})$ flavour symmetry as in [5]. We then end up with the pseudo-moduli $\theta+\theta^\star$ and the diagonal $\mathcal{X}$.

We now look for the fluctuations which give contributions to the mass matrices (4.2). They are only the ϕ_i fields, while the ξ_i and δY represent a decoupled supersymmetric sector. Indeed ξ_i and δY do not appear in bilinear terms coupled to the ϕ_i sector, so they do not contribute to the fermionic mass matrix (4.2). Even if they appear in trilinear terms coupled to the ϕ_i, they do not have the corresponding linear term:[4] they do not contribute to the bosonic mass matrix (4.2). Since $(\xi_1, \xi_2, \delta Y)$ do not couple to the breaking sector at this order, also their D-term contributions to the mass matrices vanish and all of them can be neglected. We can then restrict ourselves to the chiral ϕ_i fields for computing the 1-loop quantum corrections to the effective scalar potential using (4.2). Without loss of generality we can set the pseudo-moduli $\mathcal{X}$ proportional to the identity matrix.

The resulting superpotential for the sector affected by the supersymmetry breaking (the ϕ_i fields) is a sum of $\tilde{N}\times(N_f-\tilde{N})$ decoupled copies of a model of chiral fields which breaks supersymmetry at tree-level

$$\begin{aligned} W = & \; h_2\left(\mathcal{X}\phi_1\phi_2 - m_2^2\mathcal{X}\right) + h_2 m_2\left(e^{\theta}\phi_2\phi_5 + e^{-\theta}\phi_1\phi_6\right) + \\ & + h_1 m_2\left(e^{\theta}\phi_2\phi_3 + e^{-\theta}\phi_1\phi_4\right) + 2m_3\phi_3\phi_4 + \frac{h_1^2 m_1^2}{2m_3}\,\phi_1\phi_2 \end{aligned} \tag{4.5}$$

This superpotential doesn't have any $\mathrm{U}(1)_R$ symmetry, differently from the ones studied in [2, 5, 6]. This may be read as an example of a non generic superpotential which breaks supersymmetry [20], without exact R symmetry.

The expressions for the eigenvalues, and then for the 1-loop scalar potential, are too complicated to be written here. We can plot our results numerically to give a pictorial rapresentation.

The computation is carried out in this way: we first compute the eigenvalues of the bosonic and fermionic mass matrices (4.2) using the superpotential (4.5); we evaluate them where all the fluctuations ϕ_i are set to zero; finally we compute the 1-loop scalar potential using (4.1) as a function of the pseudo-moduli $\mathcal{X}, \theta+\theta^\star$. The corrections will always be powers of $\theta+\theta^\star \equiv \tilde{\theta}$ so from now on we will treat only the $\tilde{\theta}$ dependence. We give graphical plots of the 1-loop effective potential treating fields and couplings as real. We have checked

[4]The possible linear terms in ξ_i and δY factorize the F-equations (3.4) and so they all vanish.

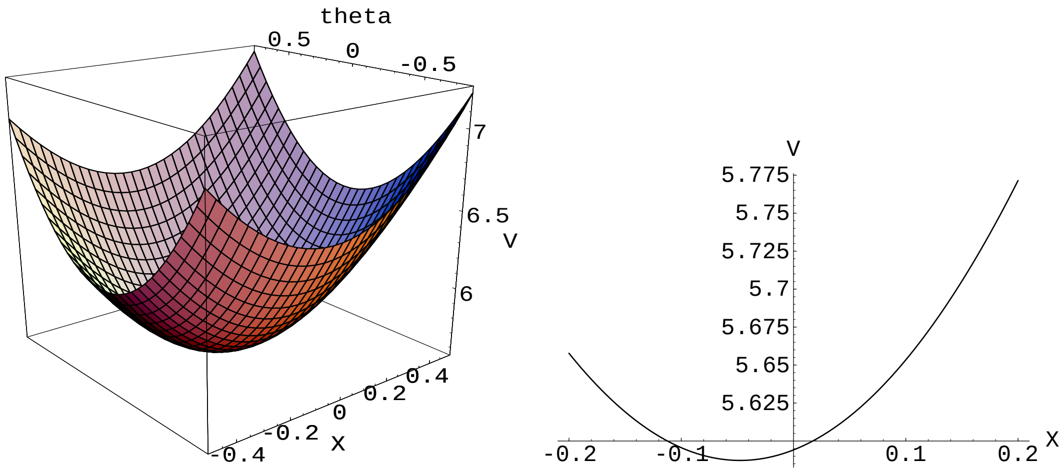

Figure 1: Scalar potential V^{1-loop} for ($\eta = 0.5, \rho = 0.1, \zeta = 0.05, \mathcal{X} = -0.5\ldots0.5, \tilde{\theta} = -0.8\ldots0.8$), and its section for $\tilde{\theta} = 0$; $\mathcal{X}$ is in unit of m_2, while V is in unit of $|h_2^2 m_2^2|^2$.

that our qualitative conclusions about the stability of the vacuum are not affected by using complex variables.

We redefine the couplings in order to have the mass matrices as functions of three dimensionless parameters (ρ, η, ζ)

$$\rho = \frac{h_1}{h_2} \qquad \eta = \frac{2m_3}{h_2 m_2} \qquad \zeta = \frac{h_1^2 m_1^2}{2h_2 m_2 m_3}\ , \qquad\qquad \zeta < \rho < \eta \tag{4.6}$$

and we rescale the superpotential with an overall scale $h_2 m_2$ which becomes the fundamental unit of our plots. The inequality in (4.6) is a consequence of the range (2.9) and the redefinitions (3.2). We notice also that ρ, η, ζ have absolute values smaller than one.

In figure 1 we plot the 1-loop scalar potential as a function of $\mathcal{X}, \tilde{\theta}$ and for fixed values of the parameters ρ, η, ζ. We can see that there is a minimum, so the moduli space is lifted by the quantum corrections, the pseudo-moduli get positive masses, and there is a stable non supersymmetric vacuum. Making a careful analysis we find that the quantum minimum in the 1-loop scalar potential is reached when $\langle\tilde{\theta}\rangle = 0$ but $\langle\mathcal{X}\rangle \neq 0$ and its vev in the minimum depends on the parameters (ρ, η, ζ). This agrees with what we observed in the previous section. It can be better seen in the second picture of figure 1 where we take a section of the first plot for $\tilde{\theta} = 0$.

In figure 2 we plot the 1-loop scalar potential for $\tilde{\theta} = 0$ as a function of $\mathcal{X}$ and of the parameter ρ, fixing η and ζ. For each value of ρ the curvature around the minimum gives a qualitative estimation of the generated mass for the pseudo-moduli $\mathcal{X}$. We note that for large ρ the scalar potential becomes asymptotically flat, and so the 1-loop generated mass goes to zero, but this is outside our allowed range.

As already observed, there is a minimum for $\langle\mathcal{X}\rangle$ slightly different from zero due to quantum corrections, and we have found that it goes to zero in the limit $(\zeta \to 0, \rho \to 0)$. We can give analytic results in this limit.[5] We found at zero order in ρ and ζ, with arbitrary

[5]Considering η, ρ, ζ real.

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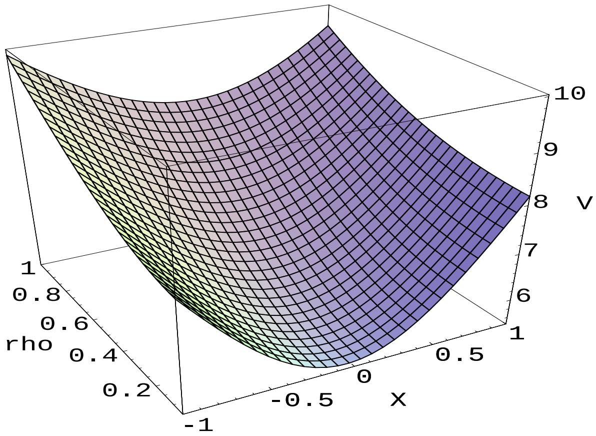

Figure 2: Scalar potential V^{1-loop} for ($\mathcal{X} = -1 \ldots 1, \rho = 0.05 \ldots 1, \eta = 0.5, \zeta = 0.05, \tilde{\theta} = 0$); $\mathcal{X}$ is in unit of m_2, while V is in unit of $|h_2^2 m_2^2|^2$.

η, that the 1-loop generated masses for the pseudo-moduli are

$$\begin{aligned} m_{\mathcal{X}}^2 &= \frac{\tilde{N}(N_f - \tilde{N})}{8\pi^2} |h_2^2 m_2|^2 (\log[4] - 1) + o(\rho) + o(\zeta) \\ m_{\tilde{\theta}}^2 &= \frac{\tilde{N}(N_f - \tilde{N})}{16\pi^2} |h_2^2 m_2^2|^2 (\log[4] - 1) + o(\rho) + o(\zeta) \end{aligned} \tag{4.7}$$

so in the limit of small ρ (and small ζ) quantum corrections don't depend on η. We can then write the 1-loop scalar potential in the limit of small ζ and ρ obtaining

$$\begin{aligned} V^{(1)} = \; & \frac{\tilde{N}(N_f - \tilde{N})}{64\pi^2} |h_2^2 m_2|^2 \Bigg\{ |m_2|^2 \Bigg(\log \big(\frac{|h_2 m_2|^2}{\Lambda^2}\big) + 2\rho^4 \log[\rho^2] - 4(1+\rho^2)^2 \log[1+\rho^2] + \\ & +2(2+\rho^2)^2 \log[2+\rho^2] \Bigg) + \Bigg(4(2+\rho^2)^2 \log[2+\rho^2] - 4\rho^4 \log[\rho^2] + \\ & -8(1+\rho^2)(1+2\log[1+\rho^2]) \Bigg) |\mathcal{X} + m_2 \zeta|^2 + |m_2|^2 \Bigg(2(1+\rho^2) \Big[(2+\rho^2)^2 \log[2+\rho^2] + \\ & -\rho^4 \log[\rho^2] - 2(1+\rho^2)(1+2\log[1+\rho^2]) \Big] + 4(\log[4] - \frac{5}{3})\zeta^2 \Bigg) (\theta + \theta^\star)^2 \Bigg\} (1 + o(\zeta)) \end{aligned} \tag{4.8}$$

In these approximations the vev for $\langle \mathcal{X} \rangle$ in the minimum is shifted linearly with ζ; however, in general, the complete behaviour for $\langle \mathcal{X} \rangle$ is more complicated and depends non trivially on η. We observe that, being ζ a simple shift for the vev of $\mathcal{X}$, it doesn't affect its mass, while it modifies $\tilde{\theta}$ mass.

From (4.8) we can read directly the masses expanding for small ρ

$$m_{\mathcal{X}}^2 = \frac{\tilde{N}(N_f - \tilde{N})}{8\pi^2}|h_2 m_2|^2 \Big(|h_2|^2(\log[4] - 1) + |h_1|^2(\log[4] - 2)\Big) \tag{4.9}$$

$$m_{\tilde{\theta}}^2 = \frac{\tilde{N}(N_f - \tilde{N})}{16\pi^2}|h_2 m_2^2|^2 \Big(|h_2|^2(\log[4] - 1) + \left|\frac{h_1^2 m_1^2}{2m_2 m_3}\right|^2 (\log[4] - \frac{5}{3}) + + |h_1|^2(2\log[4] - 3)\Big). \tag{4.10}$$

These expressions are valid up to cubic order in ρ, ζ. The first term in (4.9), (4.10), being independent of the deformations (ρ, η, ζ), agrees with [2]. The second term in (4.9) is the same as in [6].

5. Supersymmetric vacuum

Supersymmetry is restored via non perturbative effects [21], away from the metastable vacuum in the field space, when the SU($\tilde{N}$) symmetry is gauged [2]. The non supersymmetric vacuum discovered in the sections 3 and 4 is a metastable state of the theory which decays to a supersymmetric one. We are interested in evaluating the lifetime of the metastable vacuum. We need an estimation of the vevs of the elementary magnetic fields in the supersymmetric state.

We first integrate out the massive fields in the superpotential (3.1) using their equations of motion. In (3.1) there are two massive fields (M_1, Y). We integrate out the meson M_1 and the adjoint field Y tuning λ_Y in such a way that the gauge group SU($\tilde{N}$) is not broken by the adjoint,[6] as in the metastable state, so $\langle Y \rangle = 0$. Using this last condition the equation of motion for the meson M_1 gives the simple relation $M_1 = \frac{h_1}{2m_3}\left(m_1^2 - q\tilde{q}\right)$. Integrating out the charged field Y the scale matching condition reads

$$\tilde{\Lambda}^{2\tilde{N}-N_f} = \tilde{\Lambda}_{\text{int}}^{3\tilde{N}-N_f}\hat{m}_Y^{-\tilde{N}} \tag{5.1}$$

where we have indicated with $\hat{m}_Y$ the resulting mass for Y which is a combination of its tree-level mass m_Y and a term proportional to $\frac{h_3^2}{m_3}(q\tilde{q})^2$ which will be shown to be zero in the supersymmetric vacuum.

We obtain a superpotential for the meson M_2 and the flavours $(q, \tilde{q})$

$$W_{\text{int}} = \text{tr}\left(\frac{h_1^2}{4m_3}\left(2m_1^2 q\tilde{q} - (q\tilde{q})^2\right) + h_2 M_2 q\tilde{q} - h_2 m_2^2 M_2\right) \tag{5.2}$$

We expect that the supersymmetric vacua lie in the large field region, where the SU($\tilde{N}$) gauge dynamics becomes relevant [2]. We then consider large expectation value for the meson M_2. We can take as mass term for the flavours $(q, \tilde{q})$ only the vev $\langle h_2 M_2 \rangle$ neglecting the other contribution in (5.2) coming from the couplings of the magnetic theory.

[6]We are not interested in finding all the supersymmetric vacua.

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We then integrate out the flavours $(q, \tilde{q})$ using their equations of motion $(q = 0, \tilde{q} = 0)$. The corresponding scale matching condition is

$$\Lambda_L^{3\tilde{N}} = \tilde{\Lambda}_{\rm int}^{3\tilde{N}-N_f} \det(h_2 M_2) = \tilde{\Lambda}^{2\tilde{N}-N_f} \det(h_2 M_2)\, m_Y^{\tilde{N}}\,. \tag{5.3}$$

The low energy effective SU($\tilde{N}$) superpotential gets a non-perturbative contribution from the gauge dynamics related to the gaugino condensation proportional to the low energy scale Λ_L

$$W = \tilde{N}\Lambda_L^3 \tag{5.4}$$

that can be written in terms of the macroscopical scale $\tilde{\Lambda}$ using (5.3). This contribution should be added to the M_2 linear term that survives in (5.2) after having integrated out the magnetic flavours $(q, \tilde{q})$. Via the scale matching relation (5.3) we can then express the low energy effective superpotential as a function of only the M_2 meson

$$W_{\rm Low} = \tilde{N}\left(\tilde{\Lambda}^{2\tilde{N}-N_f} \det(h_2 M_2)\right)^{\frac{1}{\tilde{N}}} m_Y - m_2^2 h_2 \,\mathrm{tr}\, M_2 \tag{5.5}$$

Using this dynamically generated superpotential we can obtain the vev of the meson M_2 in the supersymmetric vacuum. Considering M_2 proportional to the identity $\mathbf{1}_{N_F}$ we minimize (5.5) and obtain

$$\langle h_2 M_2\rangle = \tilde{\Lambda}\epsilon^{\frac{\tilde{N}}{N_f-\tilde{N}}}\xi^{\frac{\tilde{N}}{N_f-\tilde{N}}}\mathbf{1}_{N_f} = m_2\left(\frac{1}{\epsilon}\right)^{\frac{N_f-2\tilde{N}}{N_f-\tilde{N}}}\xi^{\frac{\tilde{N}}{N_f-\tilde{N}}}\mathbf{1}_{N_f} \tag{5.6}$$

where

$$\epsilon = \frac{m_2}{\tilde{\Lambda}} \qquad \xi = \frac{m_2}{m_Y}\,. \tag{5.7}$$

ϵ is a dimensionless parameter which can be made parametrically small sending the Landau pole $\tilde{\Lambda}$ to infinity. ξ is a dimensionless finite parameter which doesn't spoil our estimation of the supersymmetric vacuum in the sensible range $\epsilon < \frac{1}{\xi}$. All the exponents appearing in (5.6) are positive in our window (2.5).

We observe that in the small ϵ limit the vev $\langle h_2 M_2\rangle$ is larger than the typically mass scale m_2 of the magnetic theory but much smaller than the scale $\tilde{\Lambda}$

$$m_2 \ll \langle h_2 M_2\rangle \ll \tilde{\Lambda}. \tag{5.8}$$

This fact justifies our approximation in integrating out the massive flavours $(q, \tilde{q})$ neglecting the mass term in (5.2) except $\langle h_2 M_2\rangle$. It also shows that the evaluation of the supersymmetric vacuum is reliable because the scale of $\langle h_2 M_2\rangle$ is well below the Landau pole.

6. Lifetime of the metastable vacuum

We make a qualitative evaluation of the decay rate of the metastable vacuum. At semi classical level the decay probability is proportional to e^{-S_B} where S_B is the bounce action

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from the non supersymmetric vacuum to a supersymmetric one. We have to find a trajectory in the field space such that the potential energy barrier is minimized. We remind the non supersymmetric vacuum configuration (3.6), (3.8), (3.9) and the supersymmetric one

$$q = 0 \qquad \tilde{q} = 0 \qquad Y = 0 \qquad \langle h_1 M_1 \rangle = \frac{h_1^2 m_1^2}{2m_3}\ \mathbf{1}_{N_f} \qquad \langle h_2 M_2 \rangle \neq 0 \tag{6.1}$$

where $\langle h_2 M_2 \rangle$ can be read from (5.6).

By inspection of the F-term contributions (3.4) to the potential energy it turns out that the most efficient path is to climb from the local non supersymmetric minimum to the local maximum where all the fields are set to zero but for M_1 which has the value $M_1 = \frac{h_1 m_1^2}{2m_3}\mathbf{1}_{N_f}$ as in the supersymmetric vacuum, and M_2, which is as in (3.9). This local maximum has potential energy

$$V_{\text{MAX}} = N_f |h_2 m_2^2|^2 \tag{6.2}$$

We can move from the local maximum to the supersymmetric minimum (6.1) along the M_2 meson direction. The two minima are not of the same order and so the thin wall approximation of [18] can't be used. We can approximate the potential barrier with a triangular one using the formula of [19]

$$S \simeq \frac{(\Delta\Phi)^4}{V_{\text{MAX}} - V_{\text{MIN}}} \tag{6.3}$$

We neglect the difference in the field space between all the vevs at the non supersymmetric vacuum and at the local maximum. We take as $\Delta\Phi$ the difference between the vevs of M_2 at the local maximum and at the supersymmetric vacuum. Disregarding the M_2 vev at the local maximum we can approximate $\Delta\Phi$ as (5.6). We then obtain as the decay rate

$$S \sim \left(\left(\frac{1}{\epsilon}\right)^{\frac{N_f - 2\tilde{N}}{N_f - \tilde{N}}} \xi^{\frac{\tilde{N}}{N_f - \tilde{N}}} \right)^4 \sim \left(\frac{1}{\epsilon}\right)^{4\frac{N_f - 2\tilde{N}}{N_f - \tilde{N}}} \tag{6.4}$$

This rate can be made parametrically large sending to zero the dimensionless ratio ϵ (i.e. sending $\tilde{\Lambda} \to \infty$) since the exponent $\left(4\frac{2\tilde{N} - N_f}{\tilde{N} - N_f}\right)$ is always positive in our window (2.5).

In conclusion we have found that the SU(N_c) SQCD with two adjoint chiral fields and mesonic deformations admits a metastable non supersymmetric vacuum with parametrically long life. It seems that particular care is needed in building models with adjoint matter exhibiting such vacua. The same can be said about the string geometrical construction realizing the gauge model we have studied [15, 16].

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